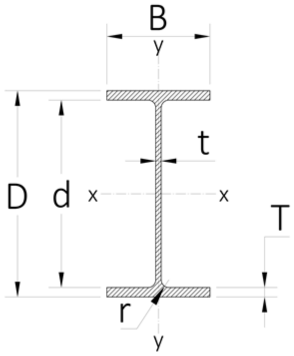
Universal beam with fixed supports and a distributed load

Dimensions and properties of structural steel EN 1993-1-1 Universal Beam:



Sizes based on British Steel Section 203 x 102 x 23 UB BS EN 10365:2017

Depth of UB section: $D := 203.2\ \text{mm}$

Width of UB section: $B := 101.8\ \text{mm}$

Flange thickness: $T := 9.3\ \text{mm}$

Web thickness: $t := 5.4\ \text{mm}$

Root radius: $r := 7.6\ \text{mm}$

Depth between flanges: $d := 184.6\ \text{mm}$

Mass of section per metre: $mass := 23.1\ \text{kg}\ \text{m}^{-1}$

Density of steel: $\rho := 7850\ \text{kg}\ \text{m}^{-3}$

Poisson's ratio in elastic range: $\nu := 0.30$

Young's modulus of elasticity: $E := 210\ \text{GPa}$

Ultimate tensile strength of hot rolled steel: $UTS := 400\ \text{MPa}$

Yield strength of hot rolled steel: $\sigma_y := 250\ \text{MPa}$

Calculations for beam loading with uniform distributed load

Length of beam: $L := 2.0\ \text{m}$

Distance to first support along beam: $L_A := 0.0\ \text{m}$

Distance to second support along beam: $L_B := 2.0\ \text{m}$

Uniform load,distance from support & load length: $F := \begin{bmatrix} 10.7\ \text{kN}\ \text{m}^{-1} \\ 0.5\ \text{m} \\ 1.0\ \text{m} \end{bmatrix}$

Factor of Safety for bending stress: $FOS := 5$

Horizontal distance from reaction point under investigation: $x := 0.75\ \text{m}$

Total weight of the beam: $w := mass \cdot L\ g_e = 0.4531\ \text{kN}$

Cross sectional area of UB section: $A := \left((2 \cdot r)^2 - \left(\frac{\pi \cdot (2 \cdot r)^2}{4} \right) \right) + 2 \cdot (B \cdot T) + (d \cdot t) = 2939.9\ \text{mm}^2$

Second moment of area on x-x axis: $I_{xx} := \left(\frac{t \cdot d^3}{12} \right) + \left(\frac{B}{12} \right) \cdot \left(D^3 - d^3 \right) = 2064.18\ \text{cm}^4$

Maximum allowable bending stress: $\sigma_a := \frac{\sigma_y}{FOS} = 50\ \text{MPa}$

Stress loading on supports

Uniform load per unit length of beam: $W = 10.7\ \mathrm{kN\ m^{-1}}$

Distance of distributed load "W" from support "A": $a = 0.50\ \mathrm{m}$

Total length of distributed load: $b = 1.00\ \mathrm{m}$

Distance of distributed load "W" from support "B": $c = 0.50\ \mathrm{m}$

Distributed load on beam: $Dist := W \cdot b = 10.7\ \mathrm{kN}$

Left support reaction force: $R1 := -\left(\frac{W \cdot b}{2 \cdot L} \cdot (2 \cdot c + b)\right) = -5.35\ \mathrm{kN}$

Right support reaction force: $R2 := -\left(\frac{W \cdot b}{2 \cdot L} \cdot (2 \cdot a + b)\right) = -5.35\ \mathrm{kN}$

Total reaction forces on beam: $R := R1 + R2 = -10.7\ \mathrm{kN}$

Distance to point load from neutral axis: $y := \frac{D}{2} = 101.6\ \mathrm{mm}$

Section modulus on x-x axis: $Z_{xx} := \frac{2 \cdot I_{xx}}{(d + 2 \cdot T)} = 203.1676\ \mathrm{cm^3}$

Normal stress on the beam: $\sigma := \frac{W \cdot L}{2 \cdot Z_{xx} \cdot L} \cdot x \cdot (L - x) = 24.6871\ \mathrm{MPa}$

Bending moment at position "x" on beam: $M_x := \frac{W \cdot x}{2} \cdot (L - x) = 5.0156\ \mathrm{kN\ m}$

Maximum bending moment of beam: $M_{max} := \frac{W \cdot L^2}{8} = 5.35\ \mathrm{kN\ m}$

Maximum bending stress on the beam: $\sigma_{max} := \frac{y \cdot W \cdot L^2}{(8 \cdot I_{xx})} = 26.3329\ \mathrm{MPa}$

Strain on the beam: $\varepsilon := \frac{\sigma_{max}}{E} = 0.000125$

if $x = \frac{L}{2}$

$$\delta_{max} := \frac{5 \cdot W \cdot L^4}{384 \cdot E \cdot I_{xx}}$$

else

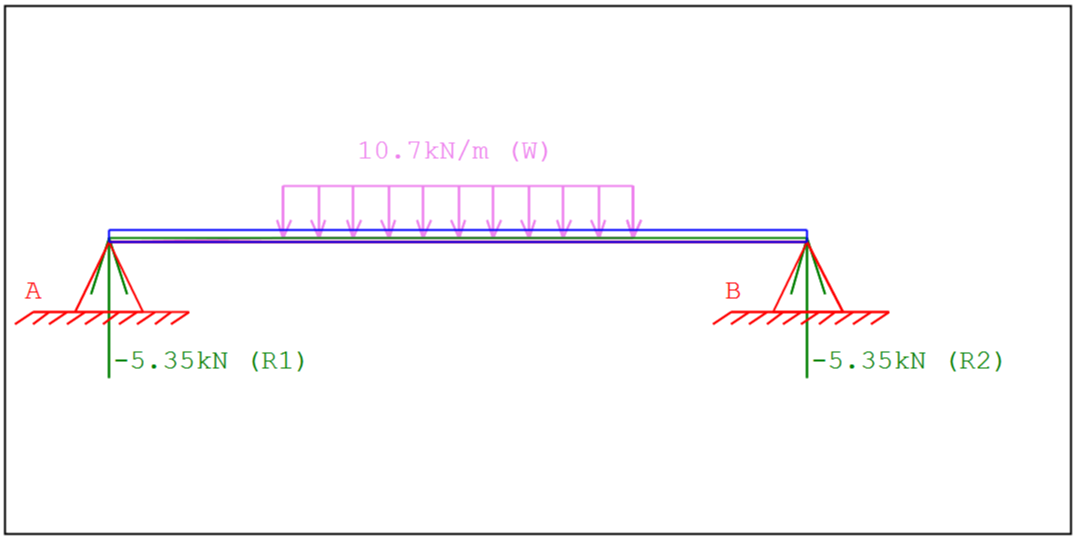
$$\delta_{max} := \frac{W \cdot x}{24 \cdot E \cdot I_{xx}} \cdot \left(L^3 - 2 \cdot L \cdot x^2 + x^3\right)$$

If-else statements to select correct equation for the Maximum deflection of the beam depending on value of x

Maximum deflection of beam: $\delta_{max} = 0.47608\ \mathrm{mm}$

Maximum extention of beam: $\Delta L := \varepsilon \cdot L = 0.2508\ \mathrm{mm}$

Diagram showing longitudinal section of beam



UB supported at both ends and a distributed load applied to beam

$result = $ "Beam unlikely to be permanently deformed"

Additional properties of steel beam not included in calculations above

Second moment of area on y-y axis: $I_{yy} := \frac{t^3 \cdot d}{12} + 2 \cdot \left(\frac{B^3 \cdot T}{12} \right) = 163.76 \text{ cm}^4$

Radius of gyration on x-x axis: $i_{xx} := \left(\frac{I_{xx}}{A} \right)^{0.5} = 8.3793 \text{ cm}$

Radius of gyration on y-y axis: $i_{yy} := \left(\frac{I_{yy}}{A} \right)^{0.5} = 2.3602 \text{ cm}$

Section modulus on y-y axis: $Z_{yy} := \frac{2 \cdot I_{yy}}{B} = 32.1736 \text{ cm}^3$

Shear modulus: $G := \frac{E}{(2 \cdot (1 + \nu))} = 80769.2308 \text{ MPa}$

Breaking stress at the extreme fibre in tension: $\sigma_{max} := \frac{M_{max}}{Z_{xx}} = 26.3329 \text{ MPa}$

Proof stress,length of plastic deformation of beam before it is permanently deformed: $\sigma_p := L \cdot 0.2 \% = 4 \text{ mm}$