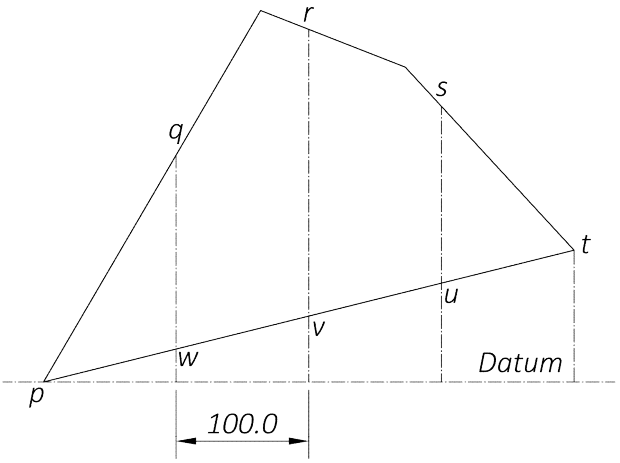
Mensuration the study of geometric figures

There are several different methods that can be used to determine the area of an irregular plane surface. The two methods below are using the simpson's rule or breaking the irregular plane into individual shapes and the use of integration to estimate the total area.

The Simpson's rule

The plane is divided into an even number of equal parts of equal width "d". The lengths of the vertical lines are measured from a datum as shown



CAD Area estimate:
53020.2854996

$d := 100.0\ mm$

$p := 0.0\ mm$

$q := 172.2222222\ mm$

$r := 267.4931129\ mm$

$s := 209.0909091\ mm$

$t := 100.00\ mm$

$u := 75.0226372\ mm$

$v := 49.9647620\ mm$

$w := 25.0\ mm$

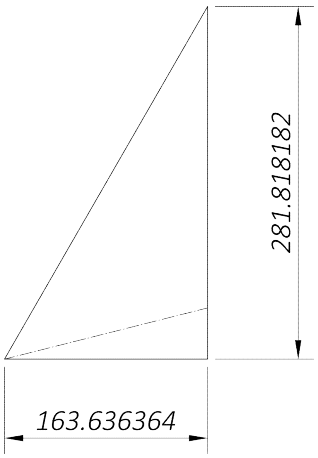
Area of irregular plane usings Simpson's rule:

Approximate area is = (d/3)x{(first+last)+4x(sum of evens)+2x(sum of odds)}

Approximate area: $A := \frac{d}{3}\cdot\left((p+t)+4\cdot(q+s)+2\cdot r\right)-\frac{d}{3}\cdot\left((p+t)+4\cdot(u+w)+2\cdot v\right) = 52007.2893\ mm^2$

Integration of area 1

The irregular plane is divided up into separate areas and the cartesian points are worked out for each area to allow the sloping line to be estimated. Integration is then used to work out the area



Cartesian points: $y_1 := 0.0\ mm$ $y_2 := 281.818182\ mm$

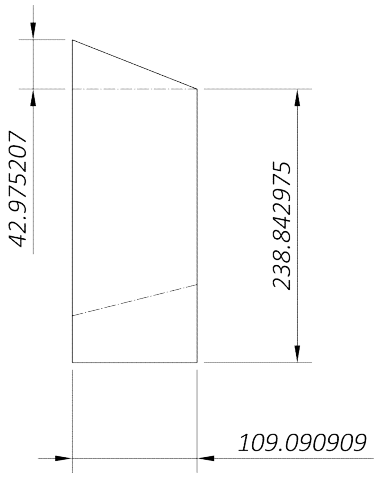
$x_1 := 0.0\ mm$ $x_2 := 163.636364\ mm$

Gradient of sloping line: $m_1 := \frac{y_2 - y_1}{x_2 - x_1} = 1.7222$

Area 1: $area_1 := \int_{x_1}^{x_2} x \cdot m_1 \, d\,x = 23057.851\ mm^2$

Length perimeter section: $p_1 := \sqrt{x_2^{\,2} + y_2^{\,2}} = 325.88088\ mm$

Integration of area 2



Cartesian points: $y_1 := 0.0\ \text{mm}$ $y_2 := 42.975207\ \text{mm}$

$x_1 := 0.0\ \text{mm}$ $x_2 := 109.090909\ \text{mm}$

$y_3 := 238.842975\ \text{mm}$

Gradient of sloping line: $m_1 := \frac{y_2 - y_1}{x_2 - x_1} = 0.3939$

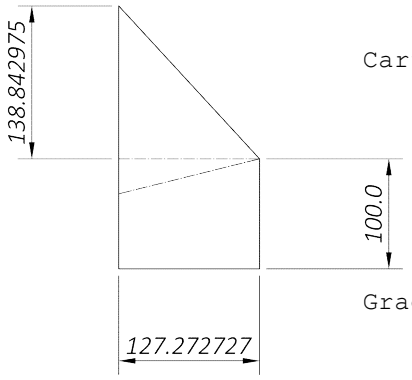
Top part of area 2: $area_{top} := \int_{x_1}^{x_2} x \cdot m_1 \, \mathrm{d}x = 2344.1022\ \text{mm}^2$

Rectangular section of area: $rec_1 := x_2 \cdot y_3 = 26055.5973\ \text{mm}^2$

Total area 2: $area_2 := area_{top} + rec_1 = 28399.699\ \text{mm}^2$

Length perimeter section: $p_2 := \sqrt{x_2^2 + y_2^2} = 117.25056\ \text{mm}$

Integration of area 3



Cartesian points: $y_1 := 0.0\ \text{mm}$ $y_2 := 138.842975\ \text{mm}$

$x_1 := 0.0\ \text{mm}$ $x_2 := 127.272727\ \text{mm}$

$y_3 := 100.0\ \text{mm}$

Gradient of sloping line: $m_1 := \frac{y_2 - y_1}{x_2 - x_1} = 1.0909$

Top part of area 3: $area_{top} := \int_{x_1}^{x_2} x \cdot m_1 \, \mathrm{d}x = 8835.462\ \text{mm}^2$

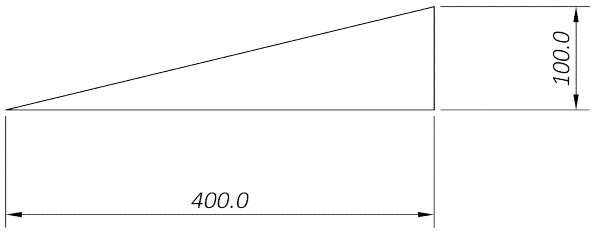
Rectangular section of area: $rec_2 := x_2 \cdot y_3 = 12727.2727\ \text{mm}^2$

Total area 3: $area_3 := area_{top} + rec_2 = 21562.7347\ \text{mm}^2$

Length perimeter section: $p_3 := \sqrt{x_2^2 + y_2^2} = 188.34999\ \text{mm}$

Integration of area 4

Area 4 is the area between the datum and the bottom of the shape that needs to be removed from the final calculation



Cartesian points: $y_1 := 0.0\ \text{mm}$ $y_2 := 100.0\ \text{mm}$

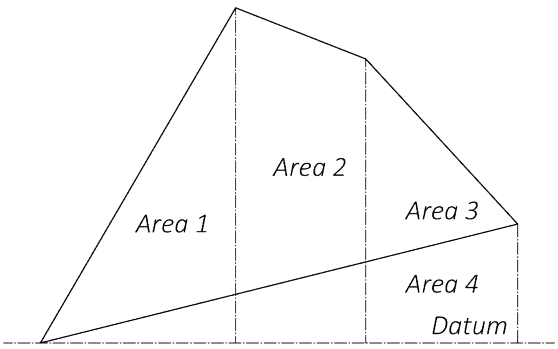
$x_1 := 0.0\ \text{mm}$ $x_2 := 400.0\ \text{mm}$

Gradient of sloping line: $m_1 := \frac{y_2 - y_1}{x_2 - x_1} = 0.25$

Total area 4: $area_4 := \int_{x_1}^{x_2} x \cdot m_1 \, \mathrm{d}x = 20000\ \text{mm}^2$

Length perimeter section: $p_4 := \sqrt{x_2^{\,2} + y_2^{\,2}} = 412.31056\ \text{mm}$

Integration of irregular plane



CAD Area estimate: 53020.2854996

From above calulations:

$area_1 = 23057.851\ \text{mm}$

$area_2 = 28399.699\ \text{mm}$

$area_3 = 21562.735\ \text{mm}$

$area_4 = 20000\ \text{mm}$

Integrated area: $A := \left(area_1 + area_2 + area_3\right) - area_4 = 53020.28548137\ \text{mm}^2$

Perimeter of irregular plane: $P := p_1 + p_2 + p_3 + p_4 = 1043.791996\ \text{mm}$