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The Development of the Empirical Formulae for River velocity

Cornelius Velsen river velocity equation based on the slope

In 1749 the Dutch hydraulics engineer, Cornelis Velsen published his great work on river management "Rivierkundige Verhandeling", (River protection discourse). He came to the conclusion that the velocity of flow should be proportional to the square root of the slope. Water does not flow on a level surface. Once the surface is raised even slightly the water starts to flow. The higher the surface is raised the faster the water flows, or to put it another way the water velocity increases. Since the slope is fundamental to allow water to flow and the angle of the slope will determine the velocity of the water.

Equations that Cornelius Velsen might have used to determine his formula in 1749.

Length of river reach used for his experiments: $L := 50\ \mathrm{m}$

Measurements obtained for vertical drop along the length: $D := 2\ \mathrm{m}$

Time it took for orange to flow downstream: $T := 4.0\ \mathrm{min}$

Pythagoras' theorem: $a^2 + b^2 = c^2$ **(eq. 1)**

Actual horizontal distance between start and end of river: $H_L := \sqrt{L^2 - D^2} = 49.96\ \mathrm{m}$ **(eq. 2)**

Velocity of river from experimental test: $V_0 := \frac{L}{T} = 0.2083\ \mathrm{m\ s^{-1}}$ **(eq. 3)**

Calculation to obtain the slope of the river: $S := \frac{D}{H_L} = 0.04$ **(eq. 4)**

Estimated velocity of river flow from Velsen equation: $V_v := \sqrt{S} = 0.2001$ **(eq. 5)**

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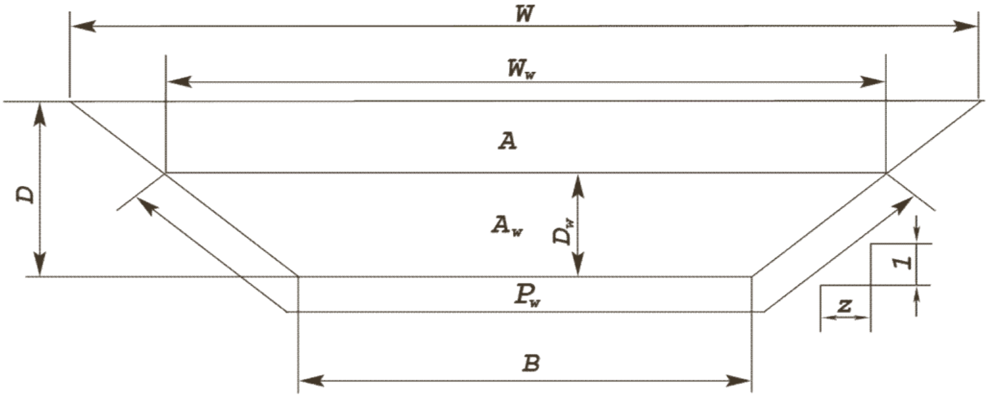
Albert Brahms Hydraulic Radius equation

Albert Brahms in 1757, considered the resistance set up was proportional to the area of cross-section divided by the length of the wetted perimeter.

If the following assumptions are made about the size and shape of the stream channel the formula suggested by Albert Brahms can be used to determine resistance of flow now known as the Hydraulic Radius:



Top width of channel: $W := 3.0\ \mathrm{m}$ Depth of the channel: $D := 0.96\ \mathrm{m}$

Bed of the channel : $B := 0.60\ \mathrm{m}$

Angle of side slope of channel in radians: $\theta := -\operatorname{atan}\left(\frac{2 \cdot D}{B - W}\right) = 0.6747\ \mathrm{rad}$ ***(eq. 6)***

Horizontal slope: $z := \frac{1}{\tan(\theta)} = 1.25$ ***(eq. 7)***

Total cross sectional area of channel: $A := B \cdot D + z \cdot D^2 = 1.728\ \mathrm{m}^2$ ***(eq. 8)***

Total wetted perimeter of channel: $P_w := B + 2 \cdot D \cdot \sqrt{1 + z^2} = 3.6735\ \mathrm{m}$ ***(eq. 9)***

The Hydraulic Radius can now be calculated:
(the Albert Brahms equation) $R := \frac{A}{P_w} = 0.4704\ \mathrm{m}$ ***(eq. 10)***

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Antoine Chezy Hydraulic velocity and Chezy coefficient equation

Antoine Chezy (1718 - 1798) a French Hydraulician is credited with the first and most lasting equation of resitance in uniform open channel flow:

Chezy Equation: $\frac{V^2}{R \cdot S}$ Therefore velocity: $V := \sqrt{R \cdot S} = 0.1372$ **(eq. 11)**

Chezy found that this value changed from one stream to another and therefore he introduced a constant C known as the Chezy Coefficient.

The Chezy formula is therefore: $C := 31$

$V := C \cdot \sqrt{R \cdot S} = 4.254$ **(eq. 12)**

It was found that the Chezy coefficient C was not a pure number but has a dimension:

$(L)^{\frac{1}{2}} \cdot (T)^{-1}$ where (L) and (T) are units of length and time of any measuring system.

The Chezy coefficient is therefore: $C := 31\ \mathrm{m}^{\frac{1}{2}}\ \mathrm{s}^{-1}$

The velocity of a stream according to Chezy formula: $V := C \cdot \sqrt{R \cdot S} = 4.254\ \mathrm{m\ s}^{-1}$ **(eq. 13)**

Pierre Louis Georges Du Buat average velocity formula

The French military engineer Pierre Louis Georges Du Buat (1734 - 1809) derived formulae for the discharge of fluids from pipes and open channels. In 1776 he begun studying hydraulics and in 1779 published the first edition of "Principes d'hydraulique". In 1786 a second volume covering experimental practice was published. In 1787 he was appointed lieutenant du roi (Lieutenant King) having risen to the rank of colonel.
During the French revolution he lost his properties and fled with his family to Belgium in 1793 and later to Germany. Du Buat proposed the formula for the average velocity.

Du Buat formula: $$V := \frac{48.85 \cdot \sqrt{R} - 0.8}{\sqrt{\frac{1}{S}} - \ln\left(\sqrt{\left(\frac{1}{S}\right)} + 1.6\right)} - 0.5 \cdot \sqrt{R} = 9.3964\ \mathrm{m\ s}^{-1}$$ **(eq. 14)**

Johann Albert Eyetelwein derived formula for open channel velocity

Johann Albert Eyetelwein (1764 - 1848) was a Prussian hydraulics engineer. After a short career in the Prussian artillery he studied civil engineering and qualified in 1790 when he left the army and entered the Prussian civil service. In 1793 he published a collection of problems in applied mathematics for surveyors and engineers. In 1801 he derived a formula for open channel velocity, which was similar to the Chézy equation, but with the Coefficient of 50.9.

Eyetelwein equation: $V := 50.9 \cdot \sqrt{R \cdot S} = 6.9848\ \mathrm{m\ s}^{-1}$ **(eq. 15)**

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The Darcy-Weisbach for head loss to frictional resistance

Julius Lugwig Weisbach (1806 - 1871) was a German mathematician who wrote 59 papers on mechanics, hydraulics, surveying and mathematics. Weisbach collaborated with Henry Darcy on the Darcy - Weisbach friction coefficient. They also developed a formula for the resistance of flow through closed pipes.

The French engineer Henry Darcy (1803 - 1858), well known for Darcy's Law, his formula for calculating the flow of water through an aquifer. In 1856 he published a report "The Public Fountains of the City of Dijon". This report included experiments with pipes that resulted in the Darcy - Weisbach friction coefficient (fD). This friction coefficient was not a constant value, but depended on the pipe diameter, roughness of the pipe wall, kinetic viscosity and velocity of the fluid flow.

Darcy-Weisbach Equation for head loss to frictional resistance: $h_L = f \cdot \frac{L}{d} \cdot \frac{V^2}{2\, g_e}$ ***(eq. 16)***

Darcy-Weisbach Equation for velocity through a closed pipe: $V = \sqrt{\frac{2 \cdot h_L \cdot d\, g_e}{f \cdot L}}$ ***(eq. 17)***

Where d = diameter of pipe, f = friction factor and L is length of pipe

By this time the resistance equation for the uniform flow in rigid bed open channels was accepted to be given by the triple factor formula:

The triple factor equation for open channel velocity: $V = C \cdot R^{x} \cdot S^{y}$ ***(eq. 18)***

Where C = coefficient, R = hydraulic radius and S0 is the slope of the river

The coefficient C and the exponents x and y were chosen to make the formula conform to the experimental data obtained by each investigator.

Henri Emile Bazin discharge coefficient (C) equation

Henri Emile Bazin (1829 - 1917) worked as an assistant to Henry Darcy. He conducted experiments to develop his hydraulic work focusing particularly on the flow of water in open channels His formula proposed in 1897 relates to the Chezy coefficient "C", the hydraulic radius and channel roughness "k". He observed that the value of "C" increased with an increase in slope, but concluded that this increase was too small to be provided for in the equation.

The Bazin roughness factor: $k := 2.246$

The Chezy discharge coefficient related to the hydraulic radius and channel roughness: $C := \frac{157.6}{1.81 + \left(\frac{k}{\sqrt{R}}\right)} = 30.9947$ ***(eq. 19)***

Where Bazin roughness factor "k" is a value between 0.109 for smooth cemented surface and about 3.2 for earthen channel in bad condition

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The Ganguillet-Kutter formula for the coefficient (C)

Emile Oscar Ganguillet (1818 - 1894) was a swiss engineer. He studied at the Progymnasium in Biel, Obergymnasium in Bern. In 1841 he worked on the construction of bridges, roads and railways in France. In 1847 he was appointed as district engineer in Delémont in Swizerland. In 1858 as Chief engineer he oversaw bridge and hydraulic structures including the Juragas water connection. In 1869 with Wilhelm Kutter he published formulae for the uniform movement of water in canals and rivers.

After an apprenticeship as a surveyor, Willhelm Rudolph Kutter (1818 – 1888), entered the service of the Kt. Bern around 1835, initially in road construction and forestry;
In 1851-88 he was secretary of the Dep. for public Buildings. Willhelm recognized the importance of a proper recording of friction losses in rivers and developed together with Cantonal Engineer Emile Ganguillet a formula for the movement of water in rivers and canals (1869 published for the first time). This friction approach for free-drain drains has been world-famous for a long time.

The Kutter roughness constant
approximately equal to Manning's "n" value: $\boxed{N := 0.029139}$

a, b and m are constants: $a := 23$ $b := 1$ $m := 0.00155$

Ganguillet-Kutter equation for constant "C":

$$C := \frac{a + \frac{b}{N} + \frac{m}{S}}{1 + \left(a + \frac{m}{S}\right) \cdot \frac{N}{\sqrt{R}}} = 28.9855 \qquad \textit{(eq. 20)}$$

Philippe Gaspard Gauckler formulae for different slope ranges

Phillipe Gaspard Gauckler (1826 - 1905), a German civil engineer trained in Strasbourg before entering the Corps of Engineers of Roads and Bridges in 1848. In 1881 he became the Chief engineer on the French Railways, until he was promoted to Inspector General of Roads and Bridges in 1886. He proposed two formulae for use in different slope ranges based on experiments by Darcy and Bazin. He also contributed to the Gauckler Manning Strickler Formula developed in 1868.

For a slope less than 0.0007: $V := \alpha \cdot R^{\frac{4}{3}} \cdot S$

and for a slope greater than 0.0007: $V := \beta \cdot R^{\frac{2}{3}} \cdot S^{\frac{1}{2}}$

(eq. 21)

For a slope less than 0.0007: $C := \alpha \cdot R^{\frac{5}{6}} \cdot S^{\frac{1}{2}}$

and for a slope greater than 0.0007: $C := \beta \cdot R^{\frac{1}{6}}$

The values of α and β are coefficients determined experimentally

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CALCULATION SHEET

The Gauckler Manning Strickler formula to determine the velocity

Albert Strickler developed Strickler scaling for the roughness of gravel bed rivers. The Strickler equation could be used to determine the Manning's rougness value "n" based on the Strickler value for the median size of the bed material in millimetres.

Assumed Strickler bed material value: $D_{50} := 0.00032$

The Strickler coefficient is related to the Manning's "n" roughness coefficient: $n := \frac{1}{K_s}$ $K_s := \frac{1}{n}$

The Manning's "n" value based on Stickler bed material value: $n := 0.132 \cdot \left(D_{50}\right)^{\frac{1}{6}} = 0.0345$ $\boxed{n := 0.0345\ \mathrm{s}\ \mathrm{m}^{-\left(\frac{1}{3}\right)}}$

The Strickler coefficient: $K_s := \frac{1}{n} = 28.9855\ \mathrm{m}^{\frac{1}{3}}\ \mathrm{s}^{-1}$

The Gauckler Manning Strickler formula: $V := K_s \cdot R^{\frac{2}{3}} \cdot S^{\frac{1}{2}} = 3.5077\ \mathrm{m}\ \mathrm{s}^{-1}$ ***(eq. 22)***

Robert Manning formula for velocity and Manning's coefficient

In 1889 Robert Manning (1816 - 1897) wrote his scientific paper "On the flow of water in open channels and pipes". The original Mannig's formula was in the paper as:

$$V := C \cdot R^{\frac{2}{3}} \cdot S^{\frac{1}{2}} \quad \textit{(eq. 23)}$$

This formula is mostly referred to as the Manning's formula, but should be more correctly called the Gauckler Manning formula. Manning later rejected this formula as it required extraction of the cube root and the equation lacked dimensional homogeneity. He proposed the following equation in his 1889 paper: (m is the pressure in metres of mercury)

$$V := C \cdot \sqrt{g_e \cdot S} \cdot \left(\sqrt{R} + \frac{0.22}{\sqrt{m}} \cdot (R - 0.15 \cdot m)\right) \quad \textit{(eq. 24)}$$

Mannings first formula however was more popular and William King (1851 - 1929) a Scottish engineer that brought about widespread acceptance of this and that Manning's coefficient "C" was the inverse of Wilhelm Rudolf Kutter's (1818 - 1888) coefficient

The coefficient "C" in the Manning's equation was written as: $C := \frac{\mu}{n}$ ***(eq. 25)***

The μ was a conversion factor, 1 for metric units and 1.485 for imperial units: $\boxed{\mu := 1}$

The Manning's roughness constant: $n = 0.0345\ \mathrm{s}\ \mathrm{m}^{-\left(\frac{1}{3}\right)}$

Mean velocity along channel at full capacity : $V := \frac{\mu}{n} \cdot R^{\frac{2}{3}} \cdot S^{\frac{1}{2}} = 3.5077\ \mathrm{m}\ \mathrm{s}^{-1}$ ***(eq. 26)***

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Summary of the Development of Empirical Formulae

Velsen formula (1749): $V := \sqrt{S} = 0.2001$ ***(eq. 5)***

Chezy formula (18c): $V := C \cdot \sqrt{R \cdot S} = 4.254\ \mathrm{m\ s}^{-1}$ ***(eq. 13)***

Du Buat formula (18c): $V := \dfrac{48.85 \cdot \sqrt{R} - 0.8}{\sqrt{\dfrac{1}{S}} - \ln\left(\sqrt{\left(\dfrac{1}{S}\right)} + 1.6\right)} - 0.5 \cdot \sqrt{R} = 9.3964\ \mathrm{m\ s}^{-1}$ ***(eq. 14)***

Eyetelwein formula (1801): $V := 50.9 \cdot \sqrt{R \cdot S} = 6.9848\ \mathrm{m\ s}^{-1}$ ***(eq. 15)***

Manning's formula (1889): $V := \dfrac{\mu}{n} \cdot R^{\frac{2}{3}} \cdot S^{\frac{1}{2}} = 3.5077\ \mathrm{m\ s}^{-1}$ ***(eq. 26)***

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